Fast Neural Networks with Circulant Projections

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Abstract

The basic computation of a fully-connected neural network layer is a linear projection of the input signal followed by a non-linear transformation. The linear projection step consumes the bulk of the processing time and memory footprint. In this work, we propose to replace the conventional linear projection with the circulant projection. The circulant structure enables the use of the Fast Fourier Transform to speed up the computation. Considering a neural network layer with d input nodes, and d output nodes, this method improves the time complexity from $\mathcal{O}(d^2)$ to $\mathcal{O}(d \log d)$ and space complexity from $\mathcal{O}(d^2)$ to $\mathcal{O}(d)$. We further show that the gradient computation and optimization of the circulant projections can be performed very efficiently. Our experiments on three standard datasets show that the proposed approach achieves this significant gain in efficiency and storage with minimal loss of accuracy compared to neural networks with unstructured projections.

1 Introduction

Deep neural network-based methods have recently achieved dramatic accuracy improvements in many areas of computer vision, including image classification [16, 30, 18], object detection [8, 21], face recognition [25, 23], and text recognition [2, 14].

These high-performing methods rely on deep networks containing millions or even billions of parameters. For example, the work by Krizhevsky *et al.* [16] achieved breakthrough results on the 2012 ImageNet challenge using a network containing 60 million parameters with five convolutional layers and three fully-connected layers. The "DeepFace" system [25] obtained face verification results close to human performance on the Labeled Faces in the Wild (LFW) dataset with a network containing 120 million parameters and a mix of convolutional, locally-connected, and fully-connected layers. If only fully-connected layers are applied, the number of parameters can grow to billions [4]. During the training stage, in order to avoid overfitting, usually millions of training samples are required to train such high-dimensional models, demanding heavy computational processing.

As larger neural networks are considered, with more layers and also more nodes in each layer, two important issues remain to be addressed:

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- How to speed-up neural network computations to meet the requirements of practical applications. Most current methods rely on GPU implementations or specialized hardware [7] to address this issue.
- How to reduce their storage cost so they can fit in memory. This is very critical, especially for embedded systems with low-memory footprint. In addition, a larger amount of parameters often means susceptibility to overfitting and, consequently, lack of generalization.

In this paper, we propose a simple, yet effective approach based on *circulant projections* to significantly reduce the computation and storage costs of fully-connected neural network layers. Notably, with much less computation and storage cost, our approach can provide very competitive error rates. In addition, the optimization process is also faster than the conventional neural networks, as well as requiring less amount of training data.

1.1 Overview of the proposed approach

A basic computation in a fully-connected neural network layer is

$$h(\mathbf{x}) = \phi(\mathbf{R}\mathbf{x}), \tag{1}$$

where $\mathbf{R} \in \mathbb{R}^{k \times d}$, and $\phi(\cdot)$ is a element-wise nonlinear activation function. The above operation connects a layer with d nodes, and a layer with k nodes. In a multi-layer perceptron, for example, all the layers are fully-connected. In convolutional neural networks, the fully connected layers are often used before the final softmax output layer, in order to capture global properties of the image. The computational complexity and space complexity of this linear projection are $\mathcal{O}(dk)$. In practice, k is usually comparable or even larger than d. This leads to computation and space complexity at least $\mathcal{O}(d^2)$, creating a bottleneck for many neural network architectures.

In this work, we propose to impose a *circulant structure* on the projection matrix $\mathbf{R}$ in (1).

$$h(\mathbf{x}) = \phi(\mathbf{R}\mathbf{x}), \quad \mathbf{R} \text{ is a circulant matrix.} \tag{2}$$

This special structure allows us to use the Fast Fourier Transform (FFT) to speed up the computation. Considering a neural network layer with d input nodes, and d output nodes, the proposed method reduces dramtically the complexity from $\mathcal{O}(d^2)$ to $\mathcal{O}(d \log d)$, and space complexity from $\mathcal{O}(d^2)$ to $\mathcal{O}(d)$. Table 1 compares the computaion and space complexity of the proposed approach with the conventional method.

Although the circulant matrix is highly structured with very small amount of parameters ($\mathcal{O}(d)$), it captures the global information well, and does not impact the final performance much. We show by experiments that our method can provide a dramatic speed up while achieving very competitive error rates.

1.2 Contributions

Our work makes the following contributions:

- We propose to impose the circulant structure on the linear projection matrix of fully-connected layers of neural networks in order to speed up computations and reduce storage costs. (Section 3)
- We propose a method which can efficiently optimize the neural network while keeping the circulant structure. (Section 4)

Method	Time	Space	Time (Learning)
Conventional NN	$\mathcal{O}(d^2)$	$\mathcal{O}(d^2)$	$\mathcal{O}(ntd^2)$
Circulant NN	$\mathcal{O}(d \log d)$	$\mathcal{O}(d)$	$\mathcal{O}(ntd \log d)$

Table 1: Comparison of the proposed method with neural networks based on unstructured projections. We assume a fully-connected layer, and the number of input nodes and number of output nodes are both d. t is the number of gradient steps in optimizing the neural network.

- We demonstrate by experiments on visual data that the proposed method can speed up the computation and reduce memory needs while maintaining competitive error rates. (Section 5)

2 Related Work

2.1 Deep Learning

In the past few years, deep neural network methods have achieved impressive results in many visual recognition tasks [16, 25, 8, 21]. Recent advances on learning these models include the use of dropout [13] for preventing overfitting, more effective non-linear activation functions such as rectified linear units [9] or max-out [10], and richer modeling through Network in Network (NiN) [18]. In particular, training high-dimensional networks with large amounts of training data has become a key success factor, but at the same time incurring increased computation and storage costs.

2.2 Speeding up Neural Networks

Several recent methods have been proposed to speed-up the computation of neural networks, with focus on convolutional architectures [14, 19, 6]. Related to our work, Mathieu et al [19] uses the Fast Fourier Transform to accelerate the computation of convolutional layers, through the well-known *Convolution Theorem*. In contrast, our work focus on the optimization of *fully-connected layers* by imposing the circulant structure on the weight matrix of each layer to speed up the computation in both training and testing stages. In addition, our method dramatically reduces the space complexity of neural networks.

Network in Network [18] has been recently proposed as a tool for richer local patch modeling in convolutional networks, where linear convolutions in each layer are replaced by convolving the input with a *micro-network* filter defined, for example, by a multi-layer perceptron. The inception architecture [24] extends this work by using these micro-networks as dimensionality reduction modules to remove computational bottlenecks and reduce storage costs. Our proposed approach could leverage similar multi-scale architectures to further improve efficiency. A key differentiating aspect is that we focus on modeling global dependencies and reducing the cost of fully connected layers, which usually contain more than 90% of parameters in standard configurations. Therefore our work is complementary to these methods. Although [18] suggests that fully-connected layers could be replaced by average pooling without hurting performance in general image classification, other work in computer vision [25] and speech recognition [22] highlight the importance of these layers to capture global dependencies and achieve state-of-the-art results.

In the context of object detection, many techniques such as detector cascades or segmentation-based selective search [26, 6] have been proposed to reduce the number of candidate object locations

in which a deep neural network is applied. Our proposed approach is complementary to these techniques. Other approaches for speeding up neural networks rely on hardware-specific optimizations. For example, fast neural network implementations have been proposed for GPUs [5], CPUs [27], and FPGAs [7].

2.3 Linear Projection with Structured Matrices

The circulant structure has been used to speed up binary embedding [29], and the Johnson-Lindenstrauss transform [1]. It has been shown that the circulant structure can be used to speed up such computational models without performance degradation.

The properties of circulant matrices have also been exploited to avoid expensive rounds of hard negative mining in training of object detectors [11] and for real-time tracking [12]. One could in principle use other structured matrices such as Hadamard matrices along with a sparse random Gaussian matrix to achieve fast projection as was done in the fast Johnson-Lindenstrauss transform [1, 3], but they are slower than the circulant projection and need more space.

3 Circulant Neural Network Model

In this section, we present the general framework of the circulant neural network model, showing the advantages of this model with respect to more efficient computational processing and storage cost savings.

3.1 Framework

A circulant matrix $\mathbf{R} \in \mathbb{R}^{d \times d}$ is a matrix defined by a vector $\mathbf{r} = (r_0, r_1, \cdots, r_{d-1})$:

$$\mathbf{R} = \mathrm{circ}(\mathbf{r}) := \begin{bmatrix} r_0 & r_{-1} & \dots & r_{-(d-2)} & r_{-(d-1)} \\ r_1 & r_0 & r_{-1} & & r_{-(d-2)} \\ \vdots & r_1 & r_0 & \ddots & \vdots \\ r_{d-2} & & \ddots & \ddots & r_{-1} \\ r_{d-1} & r_{d-2} & \dots & r_1 & r_0 \end{bmatrix}. \tag{3}$$

Let $\mathbf{D}$ be a diagonal matrix with each diagonal entry being a Bernoulli variable (± 1 with probability 1/2). For $\mathbf{x} \in \mathbb{R}^d$, its d-dimensional output is:

$$h(\mathbf{x}) = \phi(\mathbf{RDx}), \quad \mathbf{R} = \mathrm{circ}(\mathbf{r}) \tag{4}$$

The projection with the matrix $\mathbf{D}$ corresponds to a random sign flipping step on the data, which can be done as a pre-processing step. To simplify the notation, we omit this matrix in the following sections. This matrix is required in order to improve the capacity when using a circulant matrix for both binary embedding [29] and dimensionality reduction [28]. In practice, the performance of a circulant neural network (especially the randomized version) drops when the random sign flipping is not performed.

The main advantage of circulant binary embedding is its ability to use the Fast Fourier Transform (FFT) to speed up the computation. For d-dimensional data, the 1-layer circulant neural network has space complexity $\mathcal{O}(d)$, and time complexity $\mathcal{O}(d \log d)$. Next we explain why we achieve this time and space complexity.

Given a data point $\mathbf{x}$, $h(\mathbf{x})$ can be efficiently computed as follows. Denote $\circledast$ as the operator of a circulant convolution. Based on the definition of a circulant matrix,

$$\mathbf{R}\mathbf{x} = \mathbf{r} \circledast \mathbf{x}. \tag{5}$$

The convolution above can be computed more efficiently in the Fourier domain, using the Discrete Fourier Transform (DFT), for which a fast algorithm (FFT) is available.

$$h(\mathbf{x}) = \phi\left(\mathcal{F}^{-1}(\mathcal{F}(\mathbf{r}) \circ \mathcal{F}(\mathbf{x}))\right), \tag{6}$$

where $\mathcal{F}(\cdot)$ is the operator of DFT, and $\mathcal{F}^{-1}(\cdot)$ is the operator of inverse DFT (IDFT). As DFT and IDFT can be efficiently computed in $\mathcal{O}(d \log d)$ with FFT [20], the proposed approach has time complexity $\mathcal{O}(d \log d)$. Note that the circulant matrix is never explicitly computed or stored. The circulant projections are always performed by using FFT. Therefore, the storage of the model consisting of $\mathbf{r}$, and the diagonal entries of $\mathbf{D}$ takes $\mathcal{O}(d)$ space.

3.2 When $k \neq d$

We have proposed the framework of circulant neural networks considering the number of nodes in the input layer d to be equal to the number of nodes in the output layer.[1] In this section, we provide extensions to handle the case where $k \neq d$.

When $k < d$, the fully connected layer is performing a "compression" of the signal. In such a case, we still use the circulant matrix $\mathbf{R} \in \mathbb{R}^{d \times d}$ with d parameters, but the output is set to be the first k elements in (4). The circulant neural network is not computationally more efficient in this situation compared to $k = d$.

When $k > d$, the fully connected layer is performing an "expansion" of the signal. In such a case, the simplest solution is to use multiple circulant projections, and concatenate the output of them. This gives the computational complexity $\mathcal{O}(k \log d)$, and space complexity $\mathcal{O}(k)$. Note that the DFT of the feature vector can be reused in this case. An alternative approach is to extend every feature vector to a k-dimensional vector, by padding $k - d$ zeros at the end. Then the problem becomes the conventional setting as described in section 3.1 with d replaced by k. This gives space complexity $\mathcal{O}(k)$, and computational complexity $\mathcal{O}(k \log k)$. In practice, k is usually at most a few times larger than d. Empirically the two approaches give similar computational time. Our experimental results are based on the second approach.

4 Training Circulant Neural Networks

In this section, we propose a highly efficient way of training circulant neural networks. We also discuss a special type of circulant neural network, where the parameters of the circulant matrix are randomized instead of optimized.

4.1 Gradient Computation

The most critical step for optimizing a neural network given a training set is to compute the gradient of the error function with respect to the network weights. Let us consider the conventional neural network with two layers, where the first layer computes the linear projection followed by a nonlinear activation function:

$$h(\mathbf{x}) = \phi(\mathbf{R}\mathbf{x}),$$

[1] Such a setting is commonly used in fully connected layers of recent convolutional neural network architectures.

where $\mathbf{R}$ is an unstructured matrix. We assume the second layer is a linear classifier with weights $\mathbf{w}$. Therefore the output of the two-layer neural network is

$$J(\mathbf{x}) = \mathbf{w}^T \phi(\mathbf{R}\mathbf{x}) \tag{7}$$

When training the neural network, computing the gradient of the error function involves computing the gradient of $J(\mathbf{x})$ with respect to each entry of $\mathbf{R}$. It is easy to show that

$$\frac{\partial J(\mathbf{x})}{\partial R_{ij}} = w_i \phi'(\mathbf{R}_{i\cdot}\mathbf{x}) x_j, \tag{8}$$

$$i = 0, \cdots, d-1. \quad j = 0, \cdots, d-1. \tag{9}$$

where $\phi'(\cdot)$ is the derivative of $\phi(\cdot)$.

Note that (8) suffices for the gradient-based optimization of neural networks, as the gradient *w.r.t.* networks with more layers can simply be computed with the chain rule, leading to the well-known "back-propagation" scheme.

In the circulant case, we need to compute the gradient of the following objective function:

$$J(\mathbf{x}) = \mathbf{w}^T \phi(\mathbf{R}\mathbf{x}) = \sum_{i=0}^{d-1} w_i \phi\left(\mathbf{R}_{i\cdot}\mathbf{x}\right), \quad \mathbf{R} = \mathrm{circ}(\mathbf{r}).$$

It is easy to show that

$$\frac{\partial \mathbf{w}^T \phi(\mathbf{R}\mathbf{x})}{\partial r_i} = \mathbf{w}^T \left(\phi'(\mathbf{R}\mathbf{x}) \circ s_{\to i}(\mathbf{x})\right) \tag{10}$$

$$= s_{\to i}(\mathbf{x})^T (\mathbf{w} \circ \phi'(\mathbf{R}\mathbf{x})) \tag{11}$$

$s_{\to i}(\cdot) : \mathbb{R}^d \to \mathbb{R}^d$, right (downwards for a column vector) circularly shifts the vector by one element.

Therefore,

$$\begin{aligned} &\nabla_{\mathbf{r}} J(\mathbf{x}) \\ =&[s_{\to 0}(\mathbf{x}), s_{\to 1}(\mathbf{x}), \cdots, s_{\to (d-1)}(\mathbf{x})]^T (\mathbf{w} \circ \phi'(\mathbf{R}\mathbf{x})) \\ =&\,\mathrm{circ}(s_{\to 1}(\mathrm{rev}(\mathbf{x})))(\mathbf{w} \circ \phi'(\mathbf{R}\mathbf{x})) \\ =&s_{\to 1}(\mathrm{rev}(\mathbf{x})) \circledast (\mathbf{w} \circ \phi'(\mathbf{r} \circledast \mathbf{x})). \end{aligned} \tag{12}$$

where

$$\mathrm{rev}(\mathbf{x}) = (x_{d-1}, x_{d-2}, \ldots, x_0).$$

$$s_{\to 1}(\mathrm{rev}(\mathbf{x})) = (x_0, x_{d-1}, x_{d-2}, \cdots, x_1).$$

The above uses the same trick of converting the circulant matrix multiplication to circulant convolution. Therefore, computing the gradient takes only $\mathcal{O}(d \log d)$ time with FFT. Training a multi-layer neural network is nothing more than applying (11) in each layer with the chain rule.

Note that when $k < d$, we can simply set the last $d - k$ entries of $\mathbf{w}$ in (7) to be zero. And when $k > d$, the above derivations can be applied with minimal changes.

Figure 1: Sample images from the datasets used in our experimental analysis.

4.2 Randomized Circulant Neural Networks

We also consider the case where the elements of $\mathbf{r}$ in (3) are generated independently from a standard normal distribution $\mathcal{N}(0, 1)$. We refer to these models as randomized circulant neural networks. In this case, the parameters of the circulant projections are defined by random weights, without optimization. In other words, in the optimization process, only the parameters of convolutional layers and the softmax classification layer are optimized. This setting is interesting to study as it provides insight on the "capacity" of the model, independent on specific optimization mechanisms.

We will show by experiments that compared to unstructured randomized neural networks, the circulant neural network is faster with the same amount of nodes, while keeping similar performance. This surprising result is in line with the recent theoretical/empirical discoveries of using circulant projections on dimensionality reduction [28], and binary embedding [29]. It has been shown that the circulant projection behaves very similarly compared to fully randomized projections in terms of the distance preserving properties. In other words, the randomized circulant projection can be seen as a simulation of the unstructured randomized projection, both of which can capture global information of the data.

In addition, we will show that with the optimizations described in Section 4.1, the error rate of the neural networks improves significantly over the randomized version, meaning that the circulant structure is flexible and powerful enough to be used in a data-dependent fashion.

5 Experiments

We apply our model to three datasets in our experiments: MNIST, CIFAR-10, and ImageNet. Some sample images of these datasets are shown in Figure 1. We first present the implementation and the error rates of the proposed method in Sections 5.1, 5.2, and 5.3. We then comment on the computational time, space savings, and other benefits of the proposed approach in Section 5.4 and Section 5.5.

5.1 Experiments on MNIST

The MNIST digit dataset contains 60,000 training and 10,000 test images of ten handwritten digits (0 to 9), with 28×28 pixels. We use the LeNet network [17] as our basic CNN model, which is known to work well on digit classification tasks. LeNet consists of a convolutional layer followed by a pooling layer, another convolution layer followed by a pooling layer, and then two fully connected layers similar to the conventional multilayer perceptrons. We used a slightly different version from

Method	Train Error	Test Error
LeNet	0.35%	0.92%
Circulant LeNet	0.47%	0.95%

Table 2: Error rate on MNIST with LeNet.

Method	Train Error	Test Error
CIFAR-10 CNN	4.45%	15.60%
Circulant CIFAR-10 CNN	6.57%	16.71%

Table 3: Error rate on CIFAR-10 with CNN.

the original LeNet implementation, where the sigmoid activations are replaced by Rectified Linear Unit (ReLU) activations for the neurons.

Our implementation is extended from Caffe [15], by replacing the weight matrix with the proposed circulant projections with the same dimensionality. The results are compared and shown in Table 2. Our fast circulant neural network achieves an error rate of 0.95% on the full MNIST test set, which is very competitive to the 0.92% error rate from the conventional neural network.

5.2 Experiments on CIFAR

CIFAR-10 is a dataset of natural 32x32 RGB images covering 10-classes with 50,000 images for training and 10,000 for testing. Images in CIFAR-10 vary significantly not only in object position and object scale within each class, but also in object colors and textures.

The CIFAR10-CNN network [13] used in our test consists of 3 convolutional layers, 1 fully-connected layer and 1 SOFTMAX layer. Rectified linear units (ReLU) are used as the activation units. The circulant CIFAR10-CNN is implemented by adding the circulant weight matrix into the the fully connected layer. Images are cropped to 24x24 and augmented with horizontal flips, rotation, and scaling transformations. We use an initial learning rate of 0.001 and train for 700-300-50 epochs with their default weight decay.

A comparison of the error rates obtained by circulant and unstructured projections is shown in Table 3. Our efficient approach based on circulant networks obtains test error of 16.71% on this dataset, which is very competitive to the state-of-the-art.

5.3 Experiments on Imagenet (ILSVRC-2010)

ImageNet is a dataset containing over 15 million labeled high-resolution images belonging to roughly 22,000 categories. Starting in 2010, as part of the Pascal Visual Object Challenge, an annual competition called the ImageNet Large-Scale Visual Recognition Challenge (ILSVRC) has been held. A subset of ImageNet with roughly 1000 images in each of 1000 considered categories is used in this challenge. ILSVRC-2010 is the only version of ILSVRC for which the test set labels are available, so our experiments were performed on this dataset.

We use a standard CNN network - Alex-CNN [16] as the building block. The Alex-CNN network consists of 5 convolutional layers, 2 fully-connected layers and 1 final SOFTMAX layer. Rectified linear units (ReLU) are used as the activation units. Pooling layers and response normalization layers are also used between convolutional layers. Our circulant network version involves three

Method	Error Rate
CNN + Dropout	17.1 %
Circulant CNN	19.4 %
Circulant CNN + Dropout	20.3%
Randomized CNN	33.5 %
Randomized Circulant CNN	35.2%

Table 4: Classification error rate on ILSVRC-2010.

components: 1) a feature extractor, 2) fully circulant layers, and 3) a softmax classification layer. For 1 and 3 we utilize the Caffe package [15]. For 2, we implement it with Cuda FFT.

All models are trained using mini-batch stochastic gradient descent (SGD) with momentum on batches of 128 images with the momentum parameter fixed at 0.9. We train the fully connected layers using both with and without Dropout. We set the initial learning rate to 0.01, and manually decrease the learning rate if the network stops improving as in [16] according to a schedule determined on a validation set. Dataset augmentation is also exploited.

In Table 4, we show the performance of various models, comparing circulant and unstructured projections in the fully connected layers, on ILSVRC-2010. Two randomized methods have been added for comparison, where the results are shown by taking average of 10 runs. The proposed Circulant CNN achieves top-5 test set error rates of 19.4%, which is comparable to the original error rate 17.1%, with much faster training/test time, and dramatic space saving. When analyzing the two randomized methods, we can see there is large gap between them and the optimized methods. Note also that the Randomized Circulant CNN achieves similar performance to Randomized CNN. We also compare the performance of our method with Dropout and No-Dropout on the fully connected layers. We can see that circulant neural networks with the dropout strategy can not perform better than the one without dropout. This comparison indicates the power of avoid overfitting of the proposed method.

5.4 Computational Time and Storage Efficiency

As shown from the previous sections, by replacing the conventional fully connected layers with the circulant projections, we achieved very competitive performance. The main advantage of our approach is the ability to save both storage costs and computational time. In this section, we show such improvement based on a fixed hardware.

We first compare the computational cost at testing time between the circulant models and standard building block models in terms of GPU memory consumption and time cost for the entire test set on MINIST and CIFAR-10. The mini-batch size is 128. The results are summarized in Table 5. Our method provides significant improvement in both space and computation time.

We also show the training time (per image) on standard and the circulant version of Alex-CNN. We vary the number of hidden nodes d in the fully connected layers and compare the training time until the model converges (ms/per image). Table 6 shows the result. Our method provides dramatic space saving, and significant speedup compared to the conventional approach.

5.5 Reduced Training Set Size

Compared with the neural network model with unstructured projections, the circulant neural network has less parameters. This brings the benefit of good model generalization and the ability to prevent

Method	Memory(MB)	Time(sec.)
MINIST		
LeNet A	112	3.06
Circulant LeNet B	60.5	2.14
CIFAR-10		
Cifar-10 CNN	145	4.56
Circulant Cifar-10 CNN	132	3.92

Table 5: Comparisons of memory consumption and testing time cost between standard model and circulant model

d	Full projection	Circulant projection	Speedup	Space Saving
2^{10}	2.97	2.52	17.8%	1,000x
2^{12}	3.84	2.79	27.3%	4,000x
2^{14}	19.5	5.43	72.1%	30,000x

Table 6: Comparison of training time (ms/per image) and space of full projection and circulant projection. Speedup is defined as the time of circulant projection divided by the time of unstructured projection. Space saving is defined as the space of storing the circulant model by the space of storing the unstructured matrix. The unstructured projection matrix in conventional neural networks takes more than 90% of the space cost. In Alex net, d is 2^{12}.

over-fitting, especially when the number of training samples is limited. To illustrate the ability of preventing over-fitting, we reduced the training set size on MINST and CIFAR-10 datasets. We test and report the performance of each model under different training sizes. Figure 2 shows test error rate when training on a random selection of only 1,000, 2,000, 3,000, 5,000, 10,000, and half of training set. On the MNIST set, in most cases, the circulant models overfit less than the conventional approach. On CIFAR-10, this improvement is limited as the circulant layer only occupies a small part of the model.

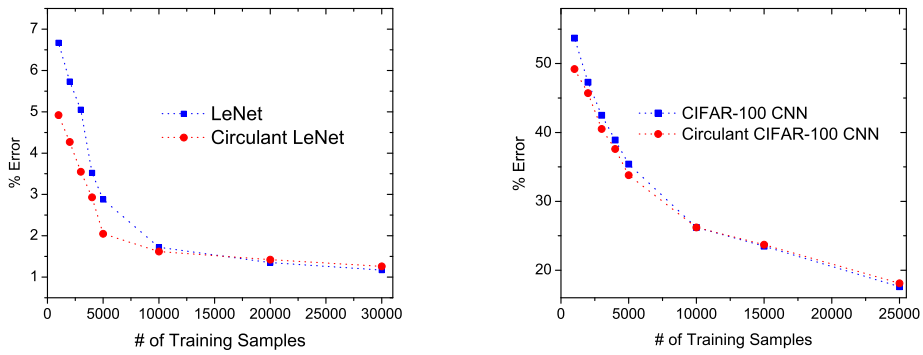
6 Discussion

6.1 Different types of neural networks

The goal of the method developed in this paper is to improve the efficiency of the fully connected layers of neural networks. These layers are used in different types of neural networks, including convolutional neural networks (CNN), and multi-layer perceptrons.

In multi-layer perceptrons, all the layers are fully connected. In convolutional architectures, the fully connected layers are often the bottleneck in terms of the space cost. For example, in "Alexnet", the fully connected layers take 90% of the storage. and roughly 20% - 30% of the computation time, based on our implementation.

One may notice that, although our approach leverages convolutions for speeding-up computations, it is fundamentally different from the convolutions performed in CNNs. The convolution filters in CNNs are all small 2D filters aiming at capturing local information of the images, whereas the proposed method is used to replace the fully connected layers, which are often "big" layers capturing global information. The circulant projection can be understood as "simulating" an unstructured



(a) MNIST (b) CIFAR-10

Figure 2: Test error when training with reduced dataset sizes of circulant CNN and conventional CNN.

projection, with much less cost. Note that one can also apply FFT to compute the convolutions on the 2D convolutional layers, but due to the computational overhead, the speed improvement is generally limited on small-scale problems. In contrast, our method can be used to dramatically speed up and scale the processing in fully connected layers. For instance, when the number of input nodes and output nodes are both 1 million, conventional linear projection is essentially impossible, as it requires TBs of memory. On the other hand, doing a convolution of two 1 million dimensional vector is a light computation task with FFT.

6.2 Towards larger neural networks

Currently, deep neural network models usually contain hundreds of millions of parameters. In real world applications, there exist problems which involve an increasing amount of data. We may need larger and deeper networks to learn better representation from large amounts of data. Compared to unstructured projections, the circulant projection significantly reduces the computation and storage cost. Therefore, with the same amount of resources, circulant neural networks can use deeper, as well as larger networks. This will likely lead to even better performance. We have conducted preliminary experiments showing that the circulant model can be extended at least 10x deeper than conventional neural networks with the same scale of computational resources.

7 Conclusions

We proposed to use circulant projections to replace the unstructured projections in order to speed up fully connected layers of neural networks. This dramatically improves the computational complexity from $\mathcal{O}(d^2)$ to $\mathcal{O}(d \log d)$ and space complexity from $\mathcal{O}(d^2)$ to $\mathcal{O}(d)$. An efficient approach was proposed for optimizing the parameters of the circulant projections. We showed by experiments that this optimization can lead to much faster convergence and training time compared to conventional neural networks. Our experimental analysis was carried out on three standard datasets, showing the effectiveness of the proposed approach. We also reported experiments on randomized circulant

projections, achieving performance similar to unstructured randomized projections. Our ongoing work includes exploring different matrix structures for circulant neural networks.

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